The University of Manchester

SPRIng MCDA© Manual

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1   Development credits

SPRIng MCDA was developed as part of the SPRIng project ("Sustainability Assessment of Nuclear Power: An Integrated Approach" funded by EPSRC and ESRC (Grant no. EP/F001444/1). The main aim of SPRIng was to develop a decision-support framework for assessing sustainability of nuclear power and other electricity options.

- Project leader: Adisa Azapagic, The University of Manchester
- Software development: Christian Barteczko-Hibbert, The University of Manchester
- Development of sustainability indicators (decision criteria): Laurence Stamford and Adisa Azapagic, The University of Manchester.

For further information on the SPRIng project visit: www.springsustainability.org. For definition of the sustainability indicators built into SPRIng MCDA see:

Stamford, L. and A. Azapagic (2011). Sustainability Indicators for the Assessment of Nuclear Power. *Energy* 36 6037-6057.

2   Starting SPRIng MCDA

SPRIng MCDA package downloaded from the SPRIng webpage contains the following modules:

- SPRIngMCDA_pkg.exe
- SPRIngMCDA.exe

The first file will install MATLAB distributable freeware, which is necessary to be able to run the SPRIng MCDA model. The second file is the SPRIng MCDA software.

1. To install MATLAB distributable:
   - Double click on SPRIngMCDA _pkg.exe
   - When the installer starts, it displays a dialog box. Read the information and then click *Next* to proceed with the installation
   - In the Folder Selection dialog box, specify the folder where you wish to install the MCR
   - Confirm your choices and click *Next*. The installer starts copying files into the installation folder
   - Click *Finish* to exit the installer.

For further detail, see the README file supplied with the SPRIng MCDA package.

If MATLAB distributable is already available on the computer used, then ensure that it is version 7.13 or later.

2. To run the SPRIng MCDA software, double click on SPRIngMCDA.exe.

3   Overview of SPRIng MCDA

Figure 1 shows the Home page which appears on opening the software (SPRIngMCDA.exe). As can be seen, two MCDA models are available:
- Fuzzy AHP and
- TOPSIS.
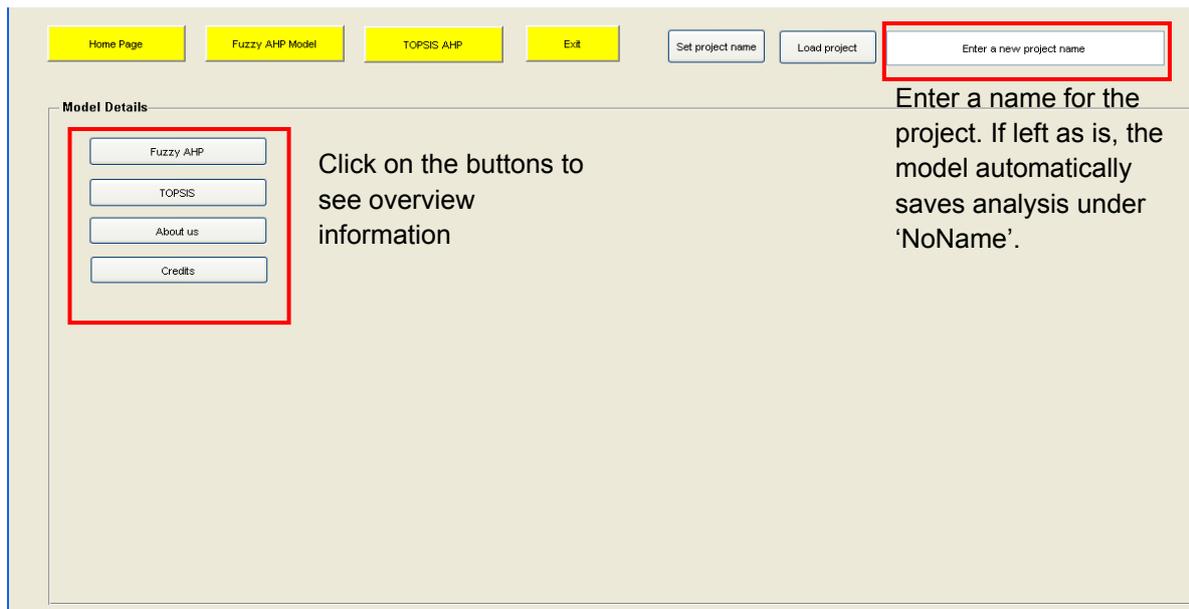
**Figure 1 The view of SPRIng MCDA on opening the software**

Clicking on the buttons *Fuzzy AHP* and *TOPSIS* buttons within the box *Model details* will open a dialogue pane, providing brief information about each MCDA model. More detailed information on the models is given in sections 3.1 and 3.2.

1. To access the individual models: click once on the yellow buttons *Fuzzy AHP* and *TOPSIS* on the top of the page. This takes the user to a new page shown in Figure 2.

2. To navigate back to the Home page or to exit the program: click once on the yellow buttons *Home page* and *Exit*, respectively.

3. To save an analysis under a specific name: enter the name in the box in the top right-hand corner, overwriting the text 'Enter a new project name'. MCDA analyses in both models are saved automatically, so there is no *Save* button.

If no name is entered, the analysis will be saved under 'NoName'. Note that any subsequent analysis for which no specific name has been entered, will also be saved under 'NoName', overwriting the previous analysis. Therefore, it is recommended to specify the name of the analysis before starting the analysis and to use different names to avoid an accidental overwriting of analyses.

All the analyses are saved in the same directory where the SPRIng MCDA software is located. There will be a series of files whose names are composed of the name given by the user and the additional name assigned by the model. For example, an analysis named Test will have a series of files named TestAlternatives, TestCriteria, etc. These are not user files, but are accessed by the software when an existing analysis is loaded up.

4. To load an existing analysis:
   - access the desired MCDA model by clicking on the yellow button *Fuzzy AHP* or *TOPSIS*
   - type the name of the analysis in the box in the top right-hand corner, overwriting the text 'Enter a new project name'.
   - click on the *Load project* button
   - click on the *Update* button (see Figure 2).
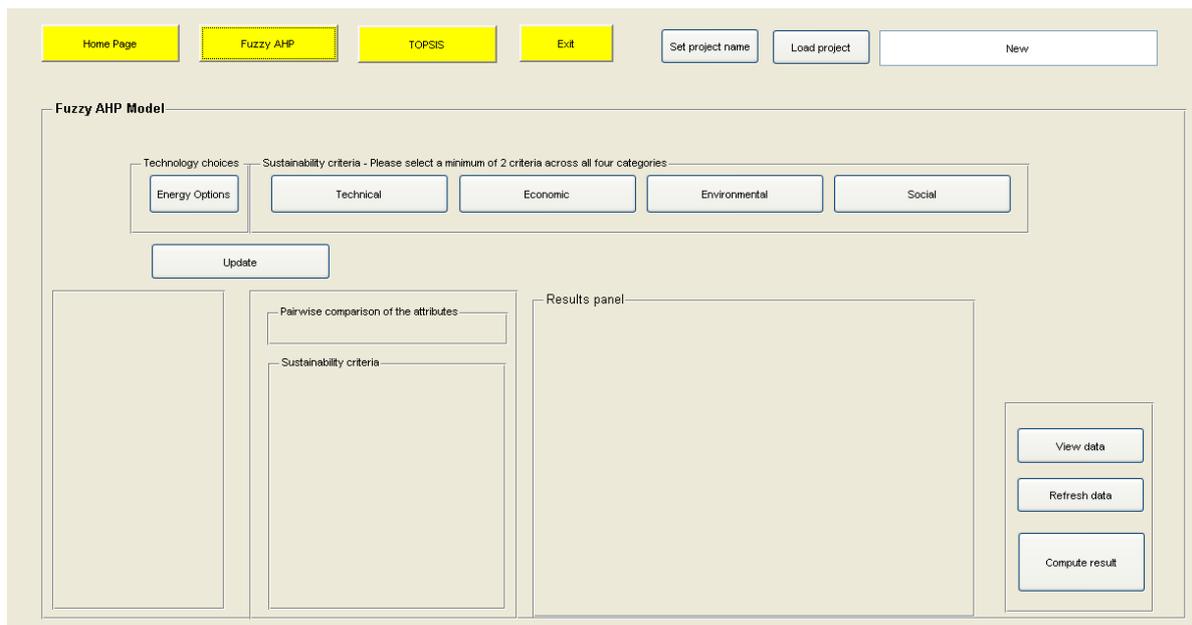
Figure 2 Top level view of the Fuzzy AHP and TOPSIS models

3.1 The fuzzy AHP model

3.1.1 *Overview*

The Analytical Hierarchy Process (AHP) is based on pairwise comparisons of different decision criteria, normally defined on a scale from 1-9, whereby 1 denotes 'least preferred' and 9 'most preferred' criterion.

Fuzzy AHP allows for ambiguity or vagueness in eliciting preferences for different decision criteria, here defined by different sustainability indicators. In addition, the decision maker (user) can specify degrees of 'confidence' and levels of 'optimism' for their preferences.

In the SPRIng fuzzy AHP method, the user is first asked to select up to five electricity options to be compared on sustainability. This is followed by selecting sustainability indicators (decision criteria) that will be used to compare the chosen electricity options. The sustainability criteria are divided into technical, economic, environmental and social; a maximum of ten criteria can be selected from the list provided. The user is then asked to pairwise compare the chosen sustainability criteria followed by pairwise comparisons of the electricity options for each criterion.

For the latter, the user can specify the levels of 'confidence' and optimism' for each comparison. As the name suggests, the confidence level shows how confident the user is in their preference of one electricity option over another for each particular decision criterion. The values range from 0 to 1, with the former denoting the lowest and 1 the highest certainty.

The level of confidence, also known as the $\alpha-cut$, is known to incorporate the experts or decision maker(s) confidence over his/her judgements. A value of 1 indicates total confidence whereas 0 suggests no confidence in the preference set. The index of optimism ranges from 0 to 1, with values below 0.5 denoting a degree of pessimism and above 0.5 indicating that the user is more optimistic with respect to their preferences. The index of optimism shows whether, when

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expressing their preference, the user is being optimistic or pessimistic – in other words, how realistic the user is in expressing their preferences and whether there is any bias.

Both the confidence and optimism parameters can be used as part of sensitivity analysis.

After computation, the result shows a ranking of the electricity options under consideration. The higher the 'overall weight', the more sustainable the option.

For the mathematical model behind the fuzzy AHP method, see the Appendix.

3.1.2 *Using the Fuzzy AHP model*

The Fuzzy AHP model can be accessed by clicking on the button *Fuzzy AHP* on the Home page. Note that it is possible to use the same analysis in both the Fuzzy AHP and TOPSIS models by clicking on the respective yellow buttons. For the TOPSIS models, see section 3.2.

To define and carry out an analysis, follow the following steps:

1. Select electricity options and the decision criteria on which these will be compared:
   * Click on the *Electricity options* button which will open a new window shown in Figure 3; up to 5 electricity options can be considered in any one analysis
   * Click on the buttons *Technical, Economic, Environmental* or *Social* to select the sustainability indicators on which the chosen electricity options will be compared. This brings up a new window; see the example shown in Figure 4. A maximum of 10 indicators can be selected.

   To see the electricity and decision criteria chosen, click the button *Update*. An example is given in Figure 5. The *Update* button can be clicked on at any point in the analysis.

   To aid the MCDA process, a table of sustainability indicators for different electricity options is provided. These can be accessed by clicking on the *View data* button, which brings up a new window (see Figure 6). The user can change these values by clicking on each value and entering a new one. Saving the table will automatically update the database for future use.

2. Carry out pairwise preferences for the criteria chosen by clicking on Criteria comparisons and choosing a preference from the drop down menu available (see Figure 7). Note that there are 9 options describing preferences in a 'fuzzy' rather than crisp way and include options such as Equal, Moderately more and Absolutely strong. For the example shown in Figure 7, Capacity factor is preferred moderately less than the Fuel cost criterion.

3. Carry out pairwise comparisons for the selected electricity options for each decision criterion by clicking on each criterion and populating the matrix (see Figure 8). At the same time, specify the levels of confidence and optimism - use either the sliding bar or enter a number between 0 and 1 to define the values.

4. Once this process has been completed for all the selected criteria, click *Compute result* to display the results. An example result is shown in Figure 5, showing a ranking graph for the options considered. A new window also pops up, showing the contribution of different decision criteria to the overall results on a graph as well as in a table (see Figure 9).

NB 1: Ensure that all the pairwise comparisons have been completed as otherwise the *Compute results* function will not work and will report an error.

NB 2: If the user wishes to change the choice of electricity options and the sustainability criteria after the pairwise comparisons have been carried out, the analysis will have to start from the beginning.
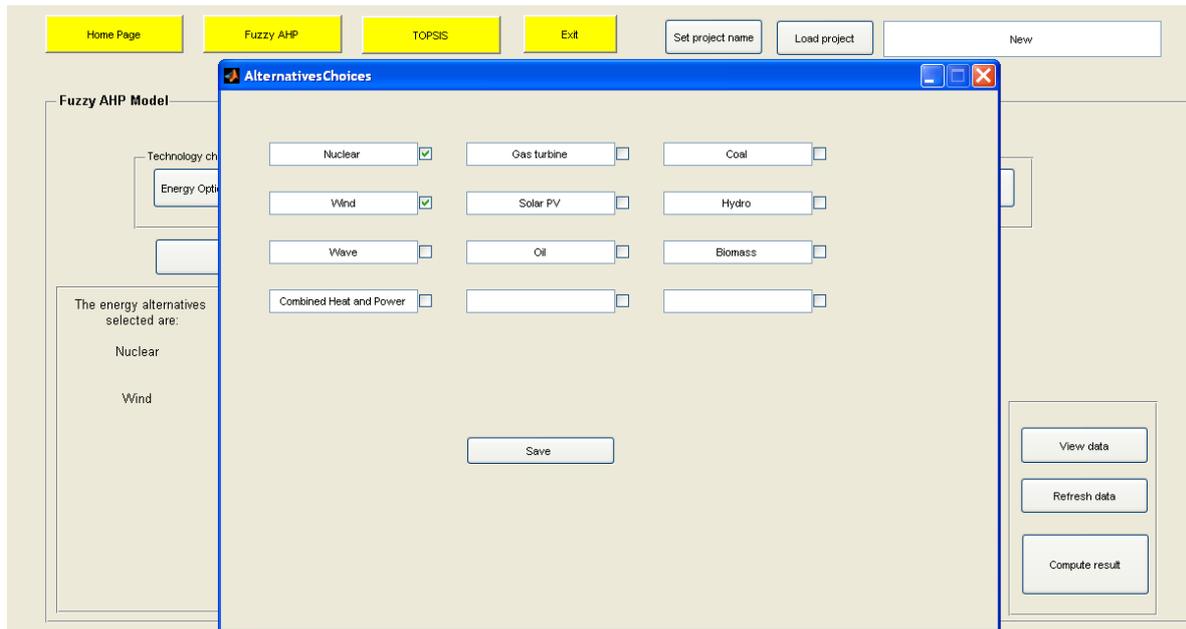

Figure 3 Choosing electricity options to be considered: click on the *Electricity options* button
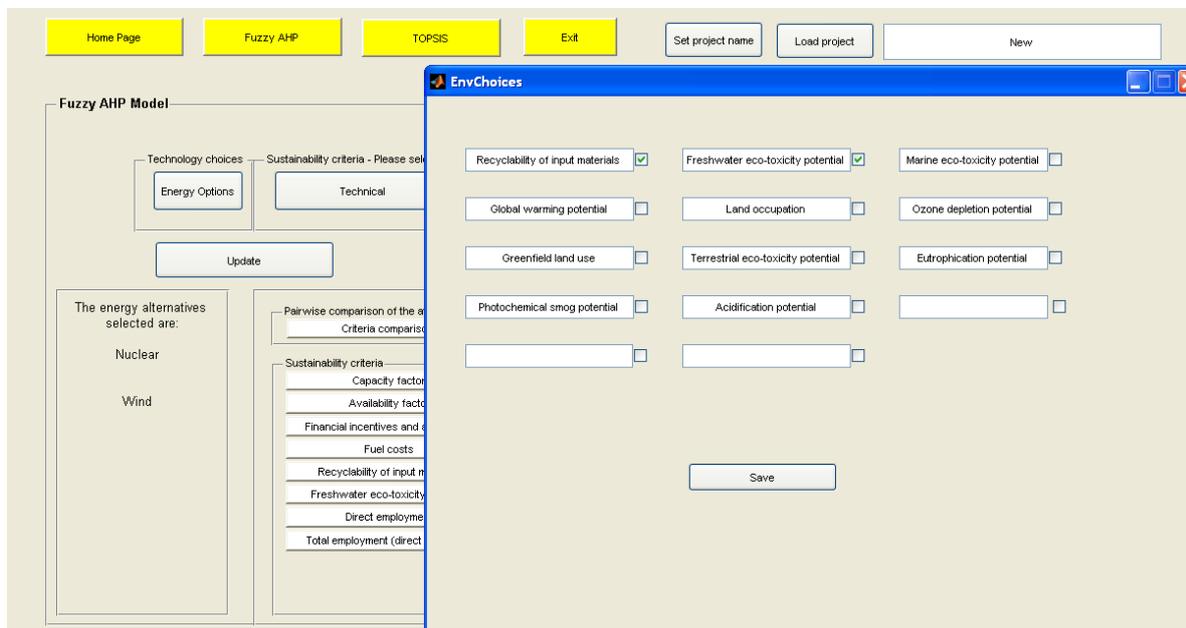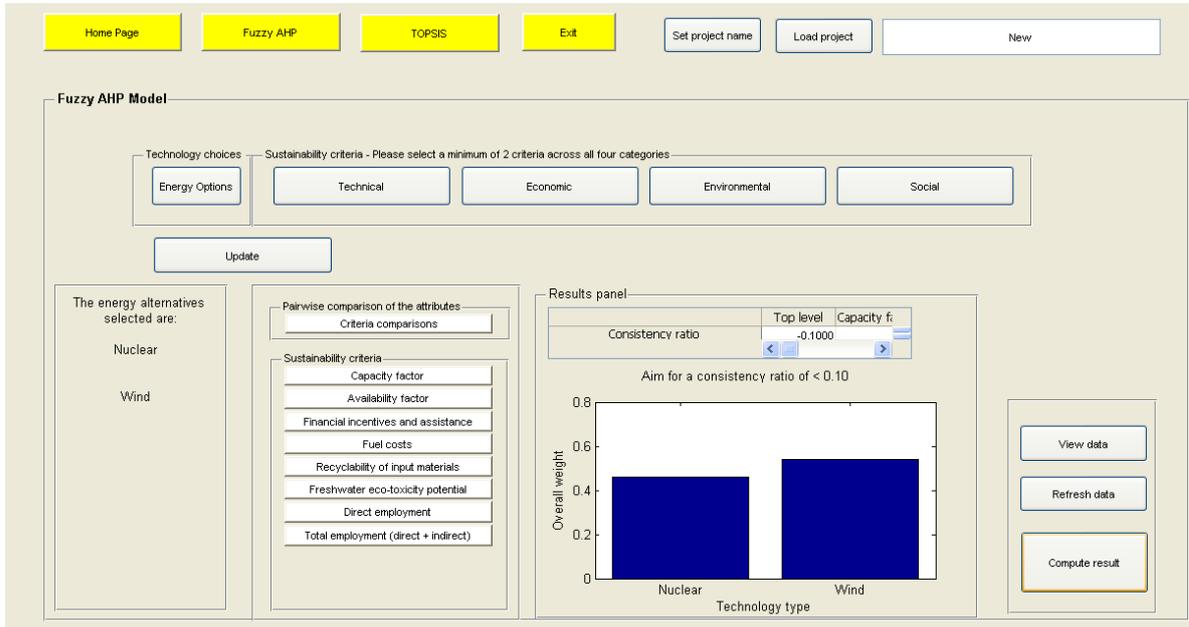

Figure 4 Choosing decision criteria (sustainability indicators) that the electricity options will be compared on: click on the *Technical, Economic, Environmental* and/or *Social indicators* button sequentially to select the decision criteria

Figure 5 Viewing the electricity options and decision criteria (sustainability indicators) selected for an analysis: click on the *Update* button. The figure also shows an example result, obtained by clicking on Compute results once all the options and criteria have been pairwise compared

#		Units	Nuclear	Gas turbine	Coal	Wind	Solar PV	Hydro	Wave
1	Capacity factor	Percentage (%)	85	62.8000	62.3000	30	8.6000	0	2
2	Availability factor	Percentage (%)	89.2000	88.7000	87.1000	81.4000	95.9000	0	
3	Technical dispatchability	Summed rank	21	22	17	28	28	0	30
4	Economic dispatchability	Dimensionless	78.2000	17.5000	54.8000	76	93.6000	0	
5	Lifetime of global fuel reserves at current extraction rates	Years	80	62.8000	119	0	0	0	0
6	Ratio of plant flexibility and operational lifetime	Years-1	1.6700	16	8.8900	0	0	0	0
7	Time to plant start-up from start of construction	Years	68	36	56	39	0.1000	0	0
8	Capital costs	Pence/kWh	64	11.4000	37.5000	108	283	0	0
9	Operation and maintenance costs	Pence/kWh	12.6000	6	11.4000	34.1000	19.4000	0	0
10	Fuel costs	Pence/kWh	5.2600	48.1000	19.7000	0	0	0	0
11	Total levelised cost	Pence/kWh	81.9000	65.4000	68.5000	142	302	0	0
12	Fuel price sensitivity	Dimensionless	6.4300	73.5000	28.7000	0	0	0	0
13	Financial incentives and assistance	Pence/kWh	5.0800	0	0	82.5000	406	0	0
14	Recyclability of input materials	Percentage (%)	0.7330	0.7940	0.7770	0.8030	0.2340	0	0
15	Freshwater eco-toxicity potential	kg 1-4 DCB‡ eq./k...	0.0024	9.3400e-05	0.0011	2.1700e-04	9.7500e-04	0	0
16	Marine eco-toxicity potential	kg 1-4 DCB‡ eq./k...	6.3600	3.5900	552	6.1000	70.4000	0	0
17	Global warming potential	kg CO_2 eq./kWh	0.0064	0.3790	1.0700	0.0118	0.0878	0	0
18	Ozone depletion potential	kg CFC-11 eq./kWh	5.4900e-10	1.2700e-08	4.2500e-09	6.3100e-10	1.7500e-08	0	0
19	Acidification potential	kg SO_2 eq./kWh	4.4700e-05	1.4800e-04	0.0018	8.7300e-05	4.3500e-04	0	0
20	Eutrophication potential	kg PO43- eq./kWh	6.3400e-06	5.8500e-05	1.8900e-04	7.4500e-06	6.4300e-05	0	0
21	Photochemical smog potential	kg C_2H_4 eq./kWh	5.0300e-06	2.4800e-05	1.4000e-04	8.5700e-06	6.3500e-05	0	0
22	Land occupation	m2yr/kWh	5.6400e-04	6.5200e-04	0.0273	6.2400e-04	0.0050	0	0
23	Greenfield land use	Percentage (%)	0.8750	0	0.1670	0	0	0	0
24	Terrestrial eco-toxicity potential	kg 1-4 DCB‡ eq./k...	3.8400e-04	1.5500e-04	0.0015	0.0012	9.4300e-04	0	0
25	Direct employment	Person-years/GWh	0.0559	0.0351	0.0556	0.3110	1.0200	0	0
26	Total employment (direct + indirect)	Person-years/GWh	0.0808	0.0624	0.1910	0.3680	0	0	0
27	Worker injuries	No. of injuries/GWh	7.1200e-04	6.7900e-04	0.0052	0.0027	0.0109	0	0
28	Human toxicity potential (excluding radiation)	kg 1-4 DCB‡ eq./k...	0.0166	0.0042	0.0690	0.0508	0.0739	0	0
29	Worker human health impacts from radiation	DALY‡/GWh	0	0	0	0	0	0	0
30	Total human health impacts from radiation (workers and po...	DALY‡/GWh	2.0400e-08	4.9500e-10	1.1100e-09	1.1800e-09	1.9900e-09	0	0
31	Fatalities due to large accidents	No. of fatalities/G...	1.2200e-09	5.0800e-06	2.0700e-05	7.7200e-07	1.1400e-06	0	0
32	Proportion of local community staff hired	Percentage (%)	0	0	0	0	0	0	0

Figure 6 Viewing the results of sustainability assessment for different electricity options and sustainability criteria: these can be accessed by clicking on the *View data* button
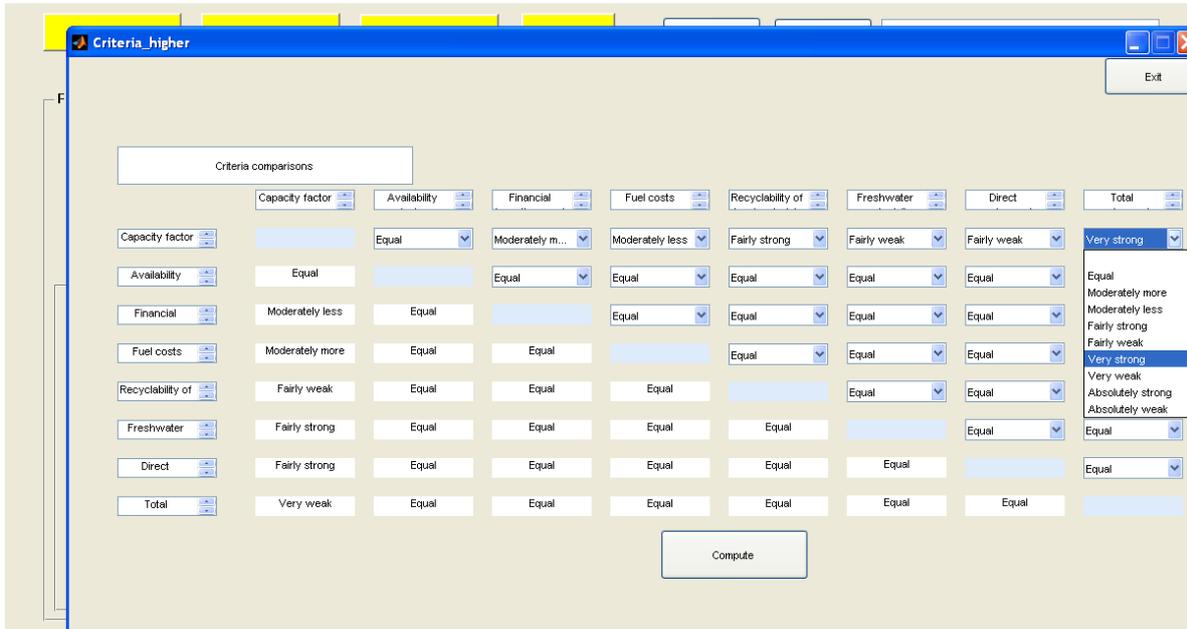
Figure 7 Pairwise comparison of selected decision criteria (sustainability indicators): click on *Criteria comparisons*
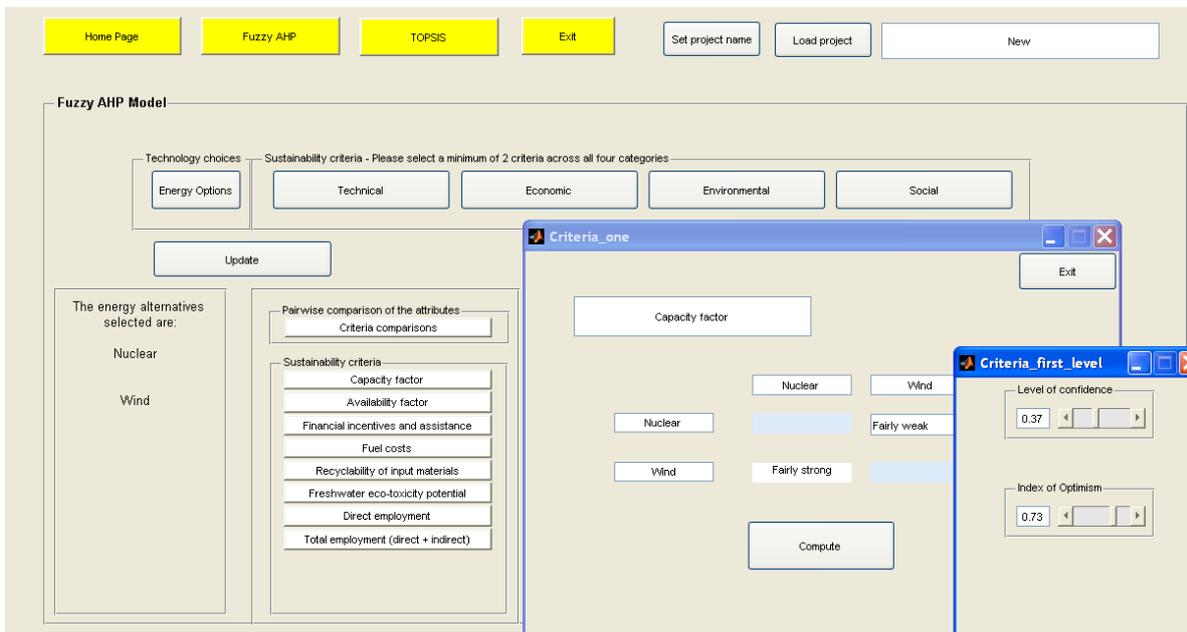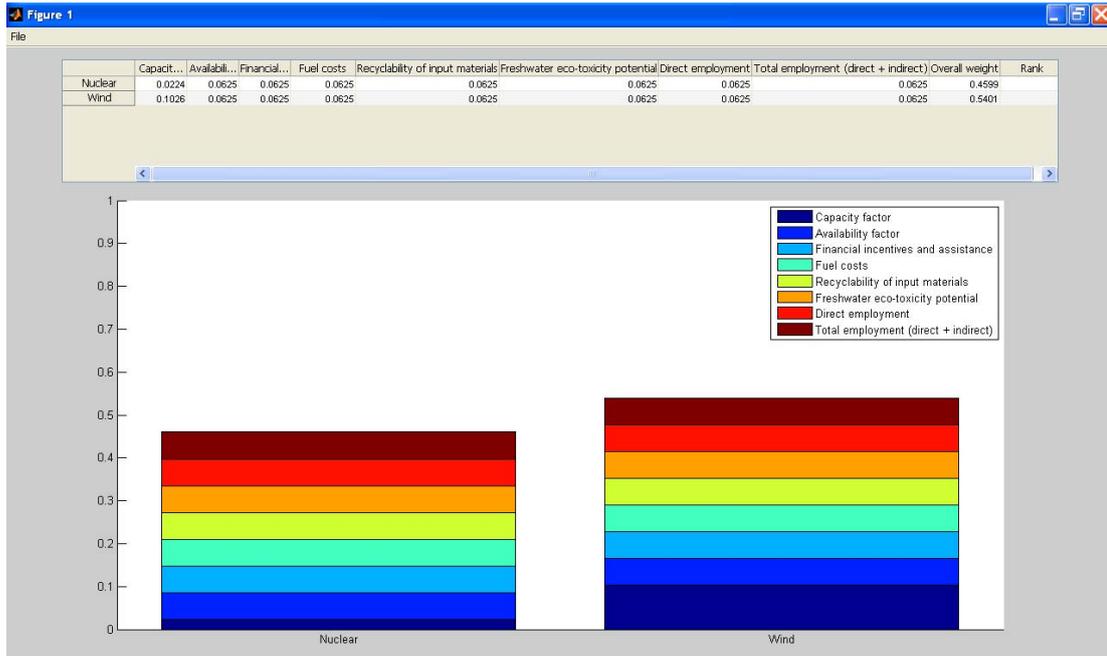

Figure 8 Pairwise comparisons of selected electricity options for a decision criterion and specification of the levels of confidence and optimism for that criterion

Figure 9 Examining the importance of different decision criteria for the overall result in the fuzzy AHP model: this can be accessed by clicking on the *Compute result* button which opens a new window with a figure as above

3.2   The TOPSIS model

3.2.1   *Overview*

The TOPSIS model is based on the closeness of an alternative to the 'ideal solution' whereby the 'best' alternative has the shortest distance from the 'positive' ideal solution and the farthest distance from the 'negative' ideal solution.

The positive ideal solution is one that maximises the 'positive' (desirable) criteria and minimises the 'negative' (undesirable) criteria, whereas the negative ideal solution does the opposite.

As in the SPRIng fuzzy AHP model, the user is first asked to select up to five electricity options to be compared on sustainability. This is followed by selecting sustainability indicators (decision criteria) that will be used to compare the chosen electricity options. The sustainability criteria are divided into technical, economic, environmental and social; a maximum of ten criteria can be selected from the list provided.

The user is then asked to pairwise compare the selected sustainability criteria and then specify whether a minimum or maximum value is preferred for each criterion. After computing, the result shows the closeness of the compared alternatives to the ideal solution – the higher the value, the more sustainable the option.

For the mathematical model behind the SPRIng TOPSIS model, see the Appendix.

3.2.2   *Using the TOPSIS model*

The TOPSIS model can be accessed by clicking on the button *TOPSIS* on the Home page. Defining an analysis in TOPSIS is exactly the same as in fuzzy AHP, expect that there is no pairwise comparison of the chosen electricity options on the sustainability criteria. Instead, the

user specifies a preference for each criterion by selecting from a drop down menu 'minimum' or 'maximum' by clicking on the *View data* button (see Figure 10).

It is possible to use the same analysis between the two models by clicking on the *Fuzzy AHP* or *TOPSIS* yellow buttons.
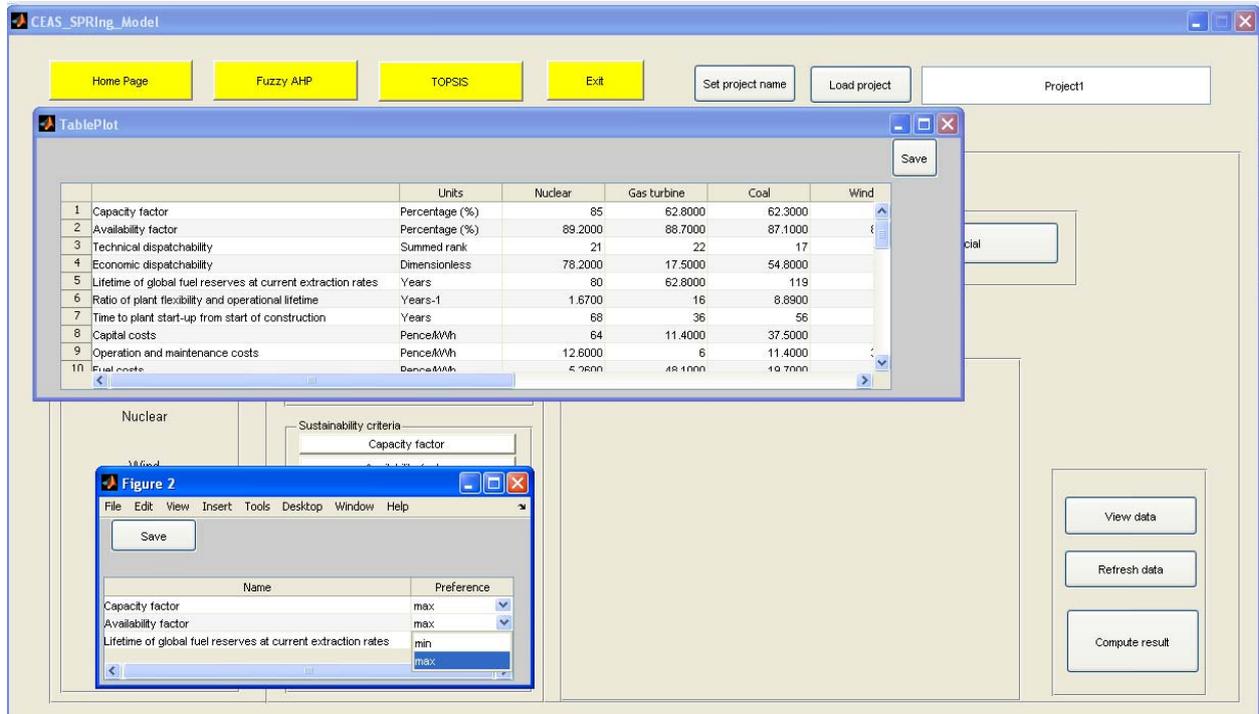

Figure 10 Viewing data in the TOPSIS model

To define and carry out an analysis using the TOPSIS model, follow the following steps:

1. Select electricity options and the decision criteria on which these will be compared:
   - Click on the *Electricity options* button which will open a new window shown in Figure 3; up to 5 electricity options can be considered in any one analysis
   - Click on the buttons *Technical, Economic, Environmental* or *Social* to select the sustainability indicators on which the chosen electricity options will be compared. This brings up a new window; see the example shown in Figure 4. A maximum of 10 indicators can be selected.

   To see the electricity and decision criteria chosen, click the button *Update*. An example is given in Figure 5. The *Update* button can be clicked on at any point in the analysis.

2. Carry out pairwise preferences for the criteria chosen by clicking on Criteria comparisons and choosing a preference from the drop down menu available (see Figure 7). Again, For the example shown in Figure 7, Capacity factor is preferred moderately less than the Fuel cost criterion.

3. Click on *View data* to specify preferences for each criterion. This opens a new window and allows choosing either 'maximum' or 'minimum' from a drop down menu (see Figure 10).

4. Once this process has been completed for all the selected criteria, click *Compute result* to display the results. An example result is shown in Figure 11, showing a ranking graph for the options considered.

NB 1: Ensure that all the pairwise comparisons have been completed as otherwise the *Compute results* function will not work and will report an error.

NB 2: If the user wishes to change the choice of electricity options and the sustainability criteria after the pairwise comparisons have been carried out, the analysis will have to start from the beginning.
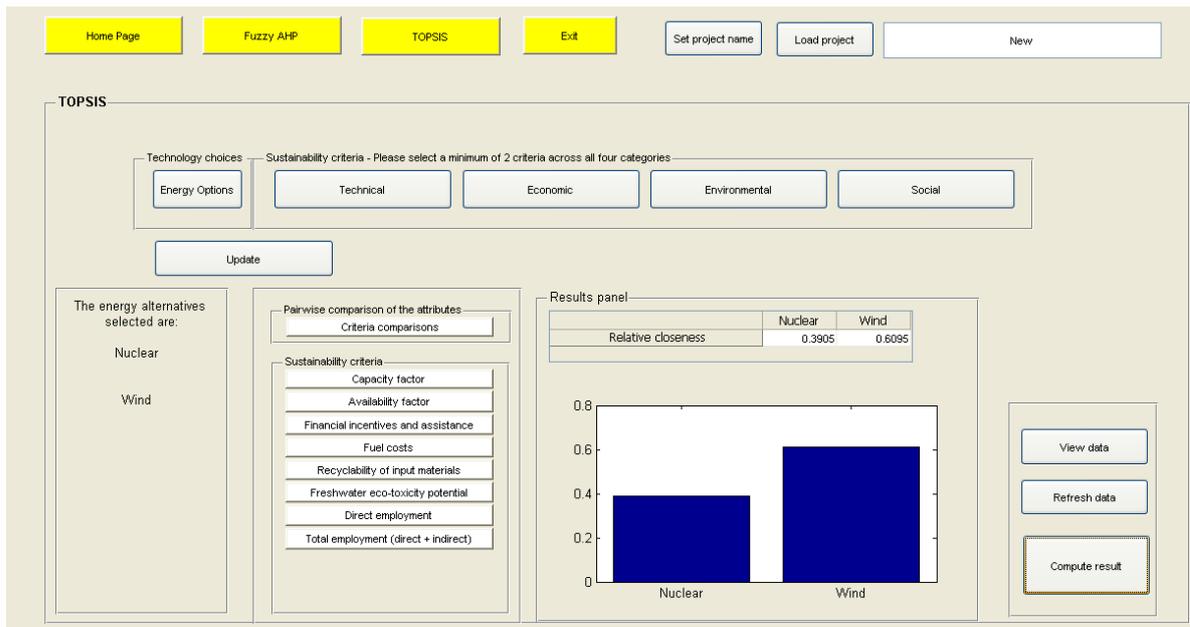

Figure 11 An example result in TOPSIS: click on Compute results to see the graph

Appendix: Mathematical models for Fuzzy AHP and TOPSIS

Fuzzy AHP

Fuzzy set theory can be utilised to help decision-makers deal with fuzzy or imprecise information. It uses a so-called 'membership function' ($\mu_{\tilde{M}}$) to indicate the extent to which one alternative is preferred to another, or the 'truth value' of that statement. Triangular fuzzy numbers $\tilde{1}$ - $\tilde{9}$ are utilised to improve the conventional AHP nine-point scheme. This is illustrated in the figure below.
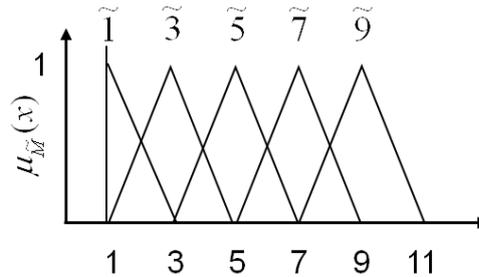
Fuzzy membership functions for the respective fuzzy numbers

A fuzzy number is a special fuzzy set $F = \{(x, \mu_F(x)), x \in R\}$, where x takes its values on the real line, $R : -\infty < x < +\infty$ and $\mu_F(x)$ is a continuous mapping from R to the closed interval [0, 1]. A triangular fuzzy number denoted as $\tilde{M} = (l, m, u)$ where $l \leq m \leq u$, has the following triangular type membership function:

$$\mu_{\tilde{M}} = \begin{cases} 0 & x < l, \\ \dfrac{x - l}{m - l} & l \leq x \leq m \\ \dfrac{u - x}{u - m} & m \leq x \leq u \\ 0 & x > u \end{cases}$$

Alternatively, by defining the interval of confidence level α, the triangular type membership function can be expressed as:

$$\tilde{M} = [l^{\alpha}, u^{\alpha}] = [(m - l)\alpha + l, -(u - m)\alpha + u]$$

Computational procedure for the fuzzy AHP algorithm:

Step 1: Compare the performance score. Linguistic terms are used to indicate the relative strength of each pair of elements in the same hierarchy. The terms in the membership function correspond to {Equal (1), Moderately more (3), Fairly strong (5), Very strong (7) Absolutely strong (9)}.

Step2: Construction of the fuzzy comparison matrix. Via pairwise comparisons of the fuzzy triangular numbers, the fuzzy judgement matrix $\tilde{A}(a_{ij})$ is constructed as given below:

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \cdots & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & 1 & \cdots & \cdots & \tilde{a}_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \cdots & 1 \end{bmatrix}$$

Where:

$$\tilde{a}_{ij}^{\alpha} = \begin{cases} 1 & i = j \\ 1, 3, 5, 7, 9 \text{ or } 1^{-1}, 1^{-1}, 3^{-1}, 5^{-1}, 7^{-1}, 9^{-1}, & i \neq j \end{cases}$$

Step 3: Solve the fuzzy eigenvalues. A fuzzy eigenvalue is a fuzzy number solution to:

$$\tilde{A}\tilde{x} = \tilde{\lambda}\tilde{x}$$

where $\tilde{\lambda}_{max}$ is the largest eigenvalue of $\tilde{A}$ and $\tilde{x}$ is a non-zero $n \times 1$ fuzzy vector containing fuzzy numbers, $\tilde{x}_i$.

The $\alpha - cut$ is known to incorporate the experts or decision makers confidence over their judgments. To perform fuzzy multiplications and additions by using the interval arithmetic and the $\alpha - cut$, the above equation is equivalent to:

$$[a_{i1l}^{\alpha} x_{1l}^{\alpha}, a_{i1u}^{\alpha} x_{1u}^{\alpha}] \oplus \cdots \oplus [a_{inl}^{\alpha} x_{nl}^{\alpha}, a_{inu}^{\alpha} x_{nu}^{\alpha}] = [\lambda x_{il}^{\alpha}, \lambda x_{iu}^{\alpha}] \, ,$$

where:

$$\tilde{A} = [\tilde{a}_{ij}], \quad \tilde{x}^t = (\tilde{x}_1, \dots, \tilde{x}_n) \qquad \tilde{a}_{ij}^{\alpha} = [a_{ijl}^{\alpha}, \quad a_{iju}^{\alpha}], \qquad \tilde{x}_i^{\alpha} = [x_{il}^{\alpha}, \quad x_{iu}^{\alpha}], \qquad \tilde{\lambda}^{\alpha} = [\lambda_l^{\alpha}, \quad \lambda_u^{\alpha}]$$

for $0 < \alpha \leq 1$ and all i, j, where $i = 1, 2, \dots n, j = 1, 2, \dots n.$

The degree of satisfaction for the judgement matrix $\tilde{A}$ is estimated by the index of optimism μ. A larger value of the index μ corresponds to a higher degree of optimism. The index of optimism is a linear convex combination defined as (Lee, 1999):

$$\tilde{a}_{ij}^{\alpha} = \mu a_{iju}^{\alpha} + (1 - \mu) a_{ijl}^{\alpha}, \qquad \forall \mu \in [0, 1]$$

When α is fixed the following matrix can be obtained after setting the index of optimism, μ, in order to estimate the degree of satisfaction:

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12}^{\alpha} & \dots & \dots & \tilde{a}_{1n}^{\alpha} \\ \tilde{a}_{21}^{\alpha} & 1 & \dots & \dots & \tilde{a}_{1n}^{\alpha} \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \tilde{a}_{n1}^{\alpha} & \tilde{a}_{n2}^{\alpha} & \dots & \dots & 1 \end{bmatrix}$$

The eigenvector is calculated by fixing the μ value and indentifying the maximal eigenvalue.

The consistency ratio (CR) is a comparison index used to control the result of the judgement matrix derived from the computed (CI) and the appropriate random consistency index (RI). The CR is defined as $CR = CI/RI$ and if less than 0.1 the comparisons are acceptable; otherwise the matrix must be re-evaluated. Using λ_{max} and knowing the dimensionality n for each fuzzy pairwise comparison matrix, the CI is computed as:

$$CI = \frac{\lambda_{max} - n}{n - 1}$$

Step 4: The priority weight of each alternative can be obtained by multiplying the matrix of evaluation ratings by the vector of attribute weights and summing over all attributes.

TOPSIS

The basic principle of TOPSIS is that the chosen alternative should have the shortest distance from some ideal solution known as the positive ideal solution whilst maximising the distance from the negative ideal solution. There are six essential steps that need to be computed in order to calculate both the positive and negative ideal solution. These are described below.

Step 1: Construct the decision matrix D to produce a ranking x_{ij} for each alternative A_i with respect to each criterion C_j: $C_1 \quad C_2 \quad \cdots \quad \cdots \quad C_n$

$$D = \begin{matrix} A_1 \\ A_2 \\ \vdots \\ \vdots \\ A_m \end{matrix} \begin{bmatrix} x_{11} & x_{12} & \cdots & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & \cdots & x_{mn} \end{bmatrix}$$

Step 2: to allow comparisons across the criteria, normalise the decision matrix:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{j=1}^{n} x_{ij}^2}}$$

where $j = 1, \ldots, n; i = 1, \ldots, m.$

Step 3: Compute the weighted normalised decision matrix v_{ij} from the weights for each criteria w_{ij} and normalised decision matrix r_{ij}:

$$V_{ij} = w_{ij} r_{ij}$$

The criteria weights are constructed solely from pairwise comparisons of the criteria themselves, so w_j is the weight of the jth criteria.

Step 4: Determine the positive ideal solution v_i^+ and the negative ideal solution, v_i^-:

$$V^+ = \{v_1^+, \ldots, v_n^+\} = \left\{ \left(\text{Max } v_{ij} \mid j \in J \right), \left(\text{Min } v_{ij} \mid j \in J' \right) \right\}$$

$$V^- = \{v_1^-, \ldots, v_n^-\} = \left\{ \left(\text{Min } v_{ij} \mid j \in J \right), \left(\text{Max } v_{ij} \mid j \in J' \right) \right\}$$

where J is associated with the positive criteria and J' is associated with the negative criteria.

Step 5: calculate the separation for each alternative using the m-dimensional Euclidean distance. The separation distance is a measure of each alternative from the positive and negative ideal solutions are given below as S_i^+ and S_i^- respectively:

$$S_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+ \right)^2}, i = 1, \ldots, m \qquad S_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^- \right)^2}, i = 1, \ldots, m$$

Step 6: calculate the relative closeness to the ideal solution and rank the alternatives in descending order. The relative closeness $\bar{C}_i$ of the alternative with respect to the positive ideal solution lies between 0 and 1 with the better performing alternative closer to unit and is given as:

$$\bar{C}_i = \frac{S_i^-}{S_i^+ + S_i^-}$$